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The spectral description of climate change including the 100 ky energy

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Abstract Core records, both ice and deep-sea, suggest that the dominant character is that of a red-noise process or random walk. Examination of a few typical records supports the inference that the contribution of the Milankovitch frequencies to climate change at most represents only a small fraction of total climate variance. Most spectral densities are sufficiently "flat" that rates of change will be dominated by the highest frequencies present in the forcing. A broad maximum near 100 ky period can be readily rationalized without invoking an oscillator. One need only suppose that there is an approximate threshold beyond which the climate system collapses. The quasiperiodicity is then governed by a combination of the collapse threshold, the system memory time scale, and the intensity of the stochastic forcing. Changes in the forcing intensity would lead to a shift in the dominant time scale. Some inferred spectral power laws may be inaccurate owing to undersampling of the records.

1 Introduction

Much of climate change study has been framed by the appearance of Milankovitch periodicities in spectral estimates of deep-sea and other cores. Beginning with the classical paper of Hays et al. (1976), great effort has been directed to studying "cyclicities" in apparent climate change, either at the Milankovitch periods or otherwise (e.g., Chapman and Shackleton 2000). Although the term "cyclicity" is not recognized by the Oxford English Dictionary, its context seems to imply climate changes approaching, if not identical to, a periodic process.

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The Milankovitch hypothesis now seems to encompass two distinct ideas: (1) the "simple" hypothesis, growing out of Hays et al. (1976), that much of climate variability occurs in the frequency bands dominating the temporal variation of solar insolation. (2) The "non-linear" hypothesis which recognizes that much of the energy in low frequency climate change occurs at periods around 100 ky, where the insolation forcing is very weak. One thus invokes the idea of the higher frequency Milankovitch forcing as a "pacemaker", controlling the 100 ky band through what is a necessarily non-linear mechanism.

The general possibility, not restricted to Milankovitch time scales, of the existence of pure frequencies in climate is an interesting and important phenomenon. Two separate issues arise: do such periodicities exist, and do they explain climate change? "Explain" is used in the sense of the percentage of the climate change variance attributable to this forcing and response: if true periodicities account for 95% of observed climate change variance, one has a very different system than one in which they account for only 5%. In the latter case, their presence is a useful tool for understanding climate change, but it is not an explanation of the major physics.

In the modern system, where there is some understanding of variability extending from periods of a few hundred years (from a few atmospheric temperature and pressure records) down to seconds and shorter, truly periodic motions are very exceptional. Apart from the ordinary tides, which are the high frequency gravitational counterpart of the Milankovitch forcing, almost no periodic motions are observed. Even the massive diurnal solar forcing of the system produces a periodic response e.g., in the wind field, which is a small fraction of the total variance. The equally massive forcing by the seasonal cycle is almost impossible to detect, e.g., in the ocean, in any physical variable below about 300 m depth. Spectra of the Southern Oscillation Index show (Wunsch 1999) a broadband character, unlike anything that might be regarded as periodic. ("Broadband" and "narrowband" are terms adopted from the signal processing literature. The latter includes both pure sinusoids, and stochastic processes with random relative phases confined to frequency bandwidths of only a few record harmonics; the former are stochastic processes involving relatively many harmonics of the record length and random phases.)

One tends to describe the modern behavior of ocean and atmosphere as a spectral continuum, on which is superimposed the special periodic motions of the tides (one example can be seen in Wunsch 1972). Here, I wish to re-examine the question of whether the much lower frequency variability of climate is not also best described as being dominated by a spectral continuum? The focus will be on time scales of 100 ky and shorter, but the question (and I believe, the answer) would apply to the entire span of climate time scales, as periodicities or "cyclicities", while not necessarily absent, are hardly the dominant mode of variability.

This point of view is an old one. A number of papers exist in the literature directed at the spectral continuum, notably Mitchell (1976) who describes the general frequency content of climate change. Pisias and Moore (1981), Shackleton and Imbrie (1990) and others produce estimates of the stochastic background in a few records and find spectral power law behavior in a number of cores. A wide literature exists e.g., on the stochastic forcing of linear and non-linear systems of varying realism. Nonetheless, comparatively little attention has been paid to the nature of this process compared to the very large literature on the apparently periodic elements. For example, a very recent paper (McDermott et al. 2001) begins with the assertion. "It is widely accepted that climate variability on time scales of 10^3 to 10^5 years is driven primarily by orbital, or so-called Milankovitch, forcing." Although not a universally shared view, it seems fair to characterize it as they do, as the one conveyed by the great majority of published papers and almost every available texbook. The consequences for the behavior of the climate components governed by the backround continuum have not often been discussed. Here, we revisit the question of the dominant nature of climate variability, and then examine the possibility that a purely stochastic system can produce an apparent dominant time scale like that observed in the major glacial cycles. The main goal here is to provide some rationalization for the red-noise character, and to show that the presence of a dominant time scale of 100 ky does not require the existence of an oscillator or of deterministic forcing.

2 Core ODP677

Since the publication of Hays et al. (1976), many more cores have been analyzed for the presence of Milankovitch cycle response. One useful example is ODP677 from 1°N, 84°W in the Panama Basin (Shackelton et al. 1990). Figures 1 and 2 display the measured δ^{18}O in the surface plankton of the core as a function of time. Figures 3, 4 are the spectral estimates $\Phi(s)$, where s is circular frequency, on logarithmic and linear scales respectively. The last 1000 ky have been used for this computation because the now familiar glacial/interglacial 100 ky cycle is visually more conspicuous there. Results for the entire record (not shown) are qualitatively similar. For purposes of this study, where fractional power is of most immediate interest, the spectra have been normalized so that the sum over all values is unity, and the accumulating sum,

$$I(s) = \sum_{n=0}^{N} \Phi(n\Delta s), \quad I(s_{\max}) = 1 \ , \tag{1}$$

is shown as a dashed line. $s_{\max}$ is the maximum frequency for which an estimate is made. A large periodic component at frequency s_1 would appear as a jump in $I(s_1)$. Δs is the frequency spacing in the estimates. The record variance, labelled σ^2, is displayed on each plot so that one could readily convert back to a spectral density, by multiplying $\Phi(s)$ by this scale factor.

Note the contrasting impressions of the logarithmic and linear axes: the latter suggests a spectrum dominated by peaks, including the obliquity band-Milankovitch forcing, and particularly one near 100 ky. The logarithmic scale display gives the impression however,

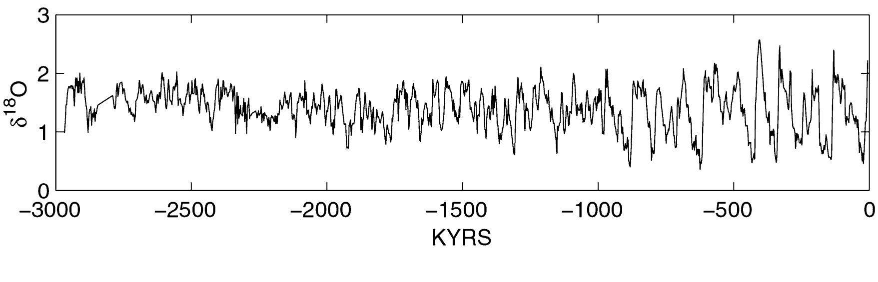


Fig. 1 Measured concentration of δ^{18}O in surface planktonics of core ODP 677 at 1.2°N, 84°W on the Costa Rica Rift; see Shackleton et al. (1990). Notice that in all figures, time runs from *left* to *right* so as to agree with conventions for modern time series

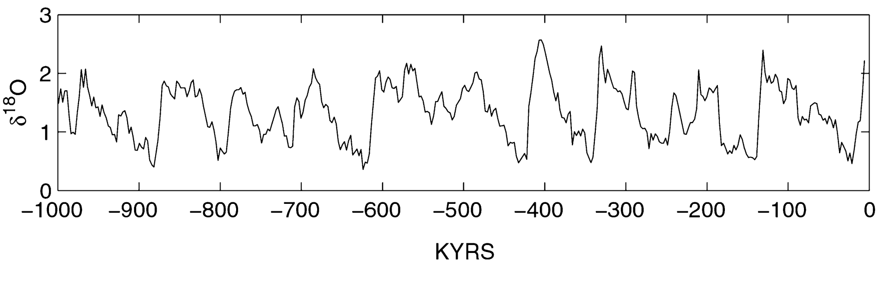


Fig. 2 Last 1 my of core ODP677 planktonic δ^{18}O (expanded from Fig. 1). Note the conspicuous non-periodic 100 ky time scale

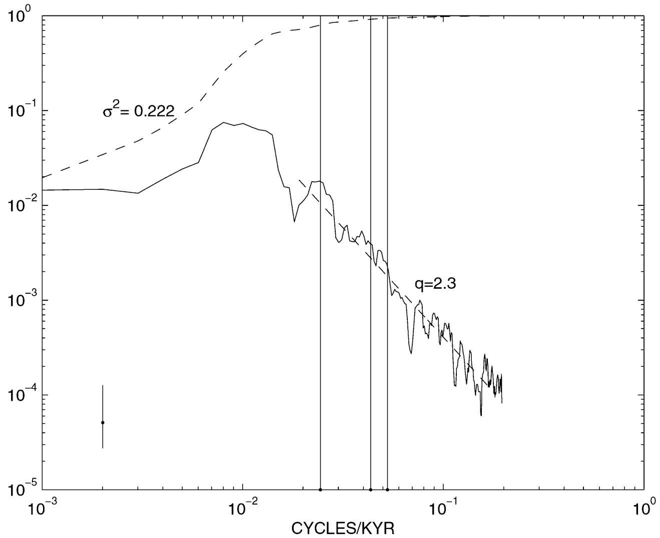


Fig. 3 Power spectrum, $\Phi(s)$, (*solid line*) normalized to sum to unity, for the record in Fig. 2, with an approximate 95% confidence interval. Dotted line is the accumulating sum, $I(s)$, of $\Phi(s)$ as a function of frequency. A least-squares fit to the power spectrum over the range of frequencies indicated by the short dashed line, of a simple power law at shorter periods results in $q = 2.3$ as shown. At low frequencies, a transition to near white noise occurs. σ^2 is the record variance in δ^{18}O units squared. Notice that both scales are logarithmic, permitting power laws to plot as *straight lines*, and the use of a constant confidence interval. Vertical lines denote periods of 41, 23 and 19 ky as a rough guide to the Milankovitch periods. The 100 ky maximum is treated here as distinct from the Milankovitch frequencies. All spectral estimates shown here are from D. Thomson's multitaper method (see Percival and Walden 1993)

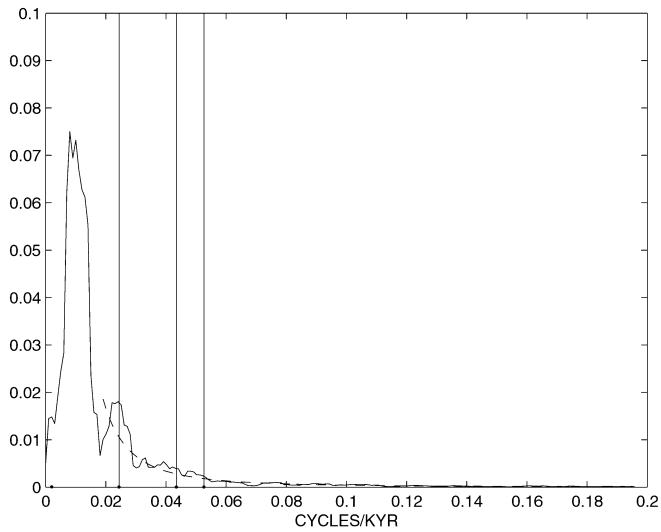


Fig. 4 Same result as in Fig. 3 except plotted on linear scales (and with the high frequencies omitted to render visible the low frequency structure). This presentation tends to emphasize the spectral peaks relative to the background continuum values, which are far more numerous, but of lower relative intensity. The 100 ky maximum is now very conspicuous. The power law plots as a *curve* on these scales

of a general red-noise process at high frequencies, on which is superimposed some weak spectral structures whose significance needs to be assessed. ("Red noise" is a stochastic time series whose energy density generally increases with decreasing frequency; it need *not* be an autoregressive process. Blue noise has an energy density increasing with frequency, and white noise has an energy density independent of frequency. The phase relationships in the Fourier transforms are random, distinguishing the behavior from deterministic processes with the same spectral shapes.) An apparent change in the character of the spectrum occurs across the 100 ky band, with the process becoming much closer to white noise at longer periods.

Visually, there is an apparent real enhancement of energy in the vicinity of the obliquity band near one cycle/41 ky (an approximate 95% confidence interval is shown). But I would assert that whatever the reality of Milankovitch or other peaks in this and other records, that the zero-order description of the spectrum of δ^{18}O would be that of red-noise continuum with a spectral shape As^{-q}, with slope, $q \approx 2$ (a fit produces $q = 2.3$, and 1.9 if the entire record is used). This same continuum dominance is visible in Figs. 6 of Hays et al. (1976), with a shape that is different in appearance caused by their use of a logarithmic abscissa and a linear ordinate.

Break the spectral density, $\Phi(s)$, into two components,

$$\Phi(s) = \Phi_c(s) + \Phi_p(s) \quad , \tag{2}$$

where $\Phi_c(s)$ is the contribution of the spectral continuum, and $\Phi_p(s)$ is that of any periodic elements including, but not necessarily limited to, the Milankovitch contribution. Spectral superposition is a consequence of assuming linearity of the system in which periodic components are simply additive to the stochastic background (e.g., Middleton 1960). Linearity is a plausible first hypothesis.

An upper bound on the importance of the Milankovitch periodicities can be obtained by assigning to them *all* of the energy lying at the Milankovitch periods, including energy more properly belonging to the background continuum. We find that,

$$\frac{\int_{M\text{-}bands} \Phi(s)\mathrm{d}s}{\int_0^{s_{\max}} \Phi(s)\mathrm{d}s} < 0.15 \quad . \tag{3}$$

Here "*M-bands*" refers to the frequency range in $\Phi(s)$ containing the bulk of the Milankovitch precessional (here taken to be everything from 26 to 18 ky periods) and obliquity (55 to 33 ky periods) energy. The bands are based upon a generous, visual, estimate of the possible region of excess energy and not upon the spectrum of insolation, which has small, but finite energy at all frequencies. Separately, the obliquity band accounts for less than 11% and the precessional band for less than 0.5% of the total variance. Recall that the record variance is, by Parseval's Theorem, equal to the integral over the entire spectral density. Similar results apply to the ODP677 benthic δ^{18}O data (not shown). These values for the contribution of the *M-band* are consistent with, but smaller than, the bound of 25% suggested by Kominz and Pisias (1979). In the spectrum of the complete record, where there is much more energy in the low frequencies, the *M-band* energy is less than 1% of the total. These values are upper bounds, as the actual energy above the background continuum is less. A more rigorous calculation would find the fraction of the core record energy coherent with the insolation, as is done in the analogous problem of tidal forcing (Munk and Cartwright 1966), but such a calculation is postponed, because of concern over the accuracy of the age/depth relations.

The 100 ky energy has sometimes been included as Milankovitch-forced, and I will deal with it more specifically later. It is worth pointing out at this stage however, that its prominence in Fig. 3 is to some extent an illusion arising from the change in overall spectral slope at this period. Consider the spectrum of the time rate of change, $\mathrm{d}\delta^{18}\mathrm{O}/\mathrm{d}t$, equivalent to multiplying $\Phi(s)$ by s^2, and shown in Fig. 5. Now the 100 ky peak is much less visually prominent. More will be said about the rate of change spectrum later. The main point is that the Milankovitch obliquity and precessional periods do not dominate the core-record variance; the fundamental characteristic of the record is that it has a continuum spectrum, with a red character and is not one dominated by pure lines.

The ODP677 record used here was not tuned to display the Milankovitch bands, having an age/depth profile fixed only at the Brunhes/Matuyama reversal at (780 ky), and Termination II at 135 ky. Consider however, the core from ODP Site 659 in the southeast North Atlantic (Fig. 6), which was *tuned* by Tiedemann et al. (1994) under the assumption that the Milankovitch precessional and obliquity frequencies dominated (the tuning details are complicated). The resulting δ^{18}O spectral density estimate in Fig. 7, from the entire record, displays the (required) Milankovitch period peaks. If one accepts the tuning hypothesis and also allocates *all* of the energy in the period bands 42 ky to 40 ky, and 24 ky to 18 ky, the resulting variance is 8% and 3% respectively, of the total. This sum of about 11% is again likely an overestimate.

3 Ubiquity of power law processes

Red-noise is probably the most common type of geophysical time series. As another independent example, but over a different range of periods, consider the spectrum of δ^{18}O from the GRIP record (GRIP Members 1993) over the last 100,000 years (Fig. 8). Again, the dominant red-noise character of the process is apparent. Slight bulges in the spectrum occur near 4000 year and 1500 year periods; the latter may well include some of the aliased annual cycle discussed by Wunsch (2000). The Vostok core (Fig. 9) is sometimes cited to show the existence in an untuned record of Milankovitch peaks. A spectral density for the Vostok CO_2 concentration (Lorius et al. 1985; Barnola et al. 1999) is very similar to the others; see Fig. 10. Figure 11 shows the periodogram of the record in Fig. 9. Three peaks, identifiable with Milankovitch periods, are indicated by the arrows; they account for only a small fraction of the record variance.

For even higher frequencies, and other variables, consider the deep-sea core, MD95-2006 from the Rockall Trough, as described by Knutz et al. (2001). This core is comparatively well-dated, with eight radiocarbon dates available in the last 39,000 years. Figure 12 shows the spectral density estimates for p-wave velocity, magnetic susceptibility, and bulk density. All are dominated by a red-noise character, but with a slope now closer to s^{-1}.

3.1 Interpretation of the spectral shape

A $q = 2$ power law spectrum at high frequencies can result from a simple one-dimensional random walk,

$$x(n\Delta t) = ax((n-1)\Delta t) + \theta(n\Delta t) \ , \tag{4}$$

with $a = 1, n = 1, 2, \ldots$, and where $\theta(n\Delta t)$ is a zero-mean pure white-noise process (wholly unpredictable). With no loss of generality, time units can be chosen so that $\Delta t = 1$. Equation (4) is a discrete analogue to a simple differential system,

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \eta(t) \ , \tag{5}$$

where t is continuous, and representing a system with an infinite memory. $\eta(t)$ is the continuous counterpart of $\theta(t)$ and would be a Brownian motion or Wiener process. If one numerically differentiates the observed time series for δ^{18}O, and recomputes the spectrum, one obtains the result in Fig. 5, near white noise at periods shorter than about 100 kyr, as expected from the approximate s^{-2} spectrum of δ^{18}O on these time scales. The dominant high frequency structure of δ^{18}O is barely distinguishable from a simple memory process, although the variations in estimated q with frequency can be readily reproduced with only slightly more complex

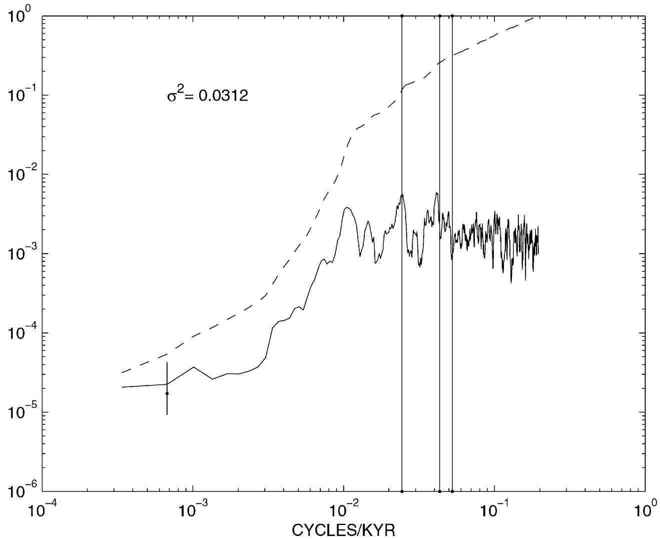
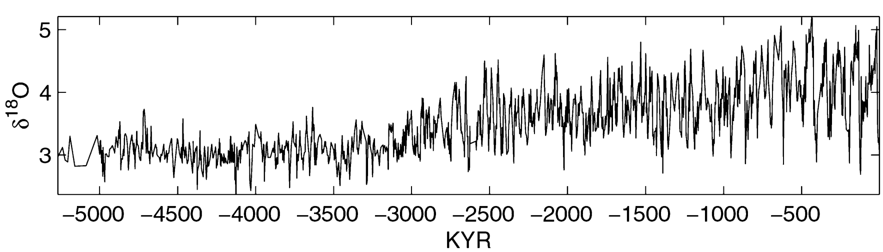


Fig. 5 Estimated spectrum of the time derivative of δ^{18}O in ODP677. Despite the structure, its main feature is a near-white character at periods shorter than 100 ky. The 100 ky energy excess is now much less conspicuous, and evidently that nature of the rate of change process changes from blue to white at about that time scale

Fig. 6 The *tuned* δ^{18}O record of Tiedemann et al. (1994)



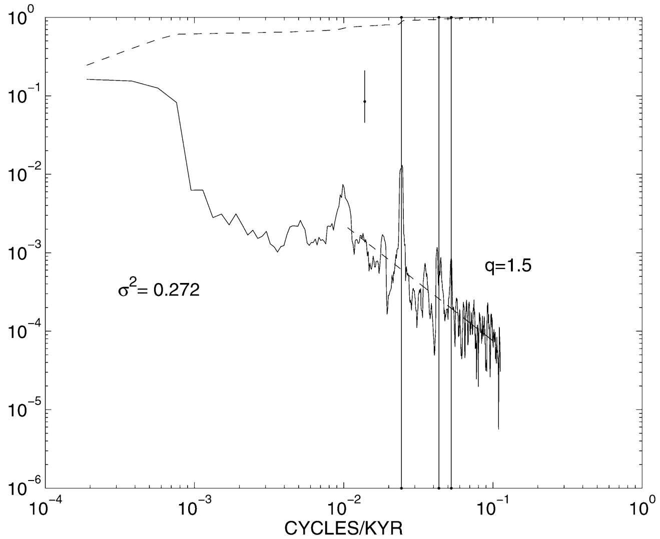


Fig. 7 Spectrum of δ^{18}O from ODP 659 as tuned by Tiedemann et al. (1994). The fraction of the record variance in the Milankovitch precessional and obliquity bands, even under this extreme hypothesis, is only about 11% of the total. The 100 ky energy is specifically omitted from this calculation; the apparent peak there is broadband in character

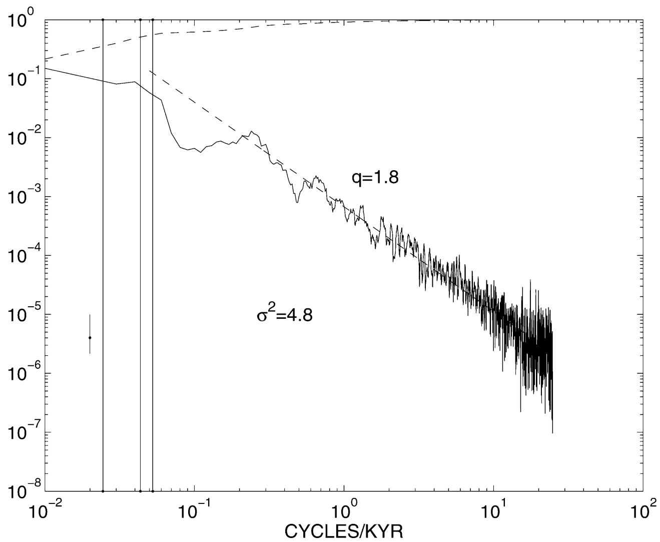


Fig. 8 Approximate power spectrum of the GRIP δ^{18}O record from 100,000 years BP and today from a linear interpolation to 300 year intervals. The power law fit produces a slope of approximately $q = 1.8$

generalizations of Eq. (4). For example, the RC11-120 record (one employed in the original Hays et al. 1976) analysis, is reproduced quite accurately as an autoregressive process,

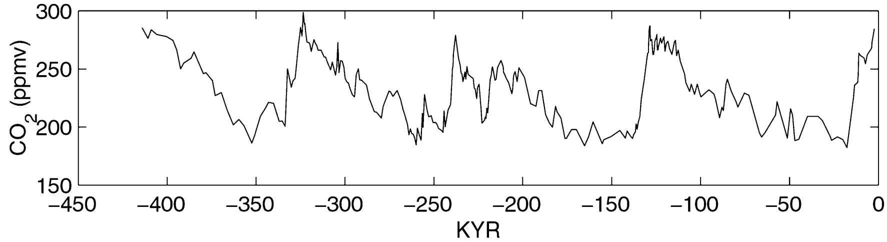
$$\begin{aligned} x(n\Delta t) = {} & 1.11x((n-1)\Delta t) + 0.056x((n-2)\Delta t) \\ & -0.11x((n-3)\Delta t) \\ & -0.041x((n-4)\Delta t) - 0.28x((n-5)\Delta t) \\ & +0.18x((n-6)\Delta t) + \theta(n\Delta t), \quad \Delta t = 1.6\,\mathrm{ky} \ , \end{aligned} \tag{6}$$

and it is straightforward to convert this representation into an equivalent differential system. Deviations from $q = 2$ can arise from other causes including undersampling (taken up later), suppression of some spatial/temporal scales by the ocean circulation, bioturbation, variations in the sedimentation rate, and the summation of independent processes controlling, e.g., bulk density.

The spectrum of the time rate of change of physical variables is of intense interest in its own right: usually governing equations describe not a physical variable such as temperature, but rather their temporal rates of change or even their second derivatives in terms of e.g., heat sources. If the underlying physical parameter has a reddish spectrum, its rate of change will tend to have a white or even blue spectrum, showing that the apparently very small energy levels at high frequencies may in fact be of equal or greater importance to the physics than the more energetic low frequencies. Because differentiation multiplies spectra by s^2, the transition from a rate-of-change spectrum that is also red, to one that is blue occurs when $\Phi(s) \propto s^{-2}$.

Equation (5) was used by Hasselmann (1976) in his stochastic theory of climate and by many later investigators. It appears that much of the climate system is best understood in terms of such random walks (e.g., Wunsch 1992 for the ocean; Mitchell 1976 for the climate system in general, and a large specific literature for the modern climate system). Arbitrarily more complex stochastic memory processes can be generated by using higher derivatives in Eq. (5) and of course, by recognizing that spatially varying structures will also be present and contribute. More complex discrete representations are available, and there is an elaborate literature on stochastic differential systems, e.g., Gardiner (1985), or Gillespie (1996). For our present purposes, the discrete representation is adequate.

Fig. 9 Carbon dioxide concentration in the Vostok Antarctic ice core (Barnola et al. 1999)



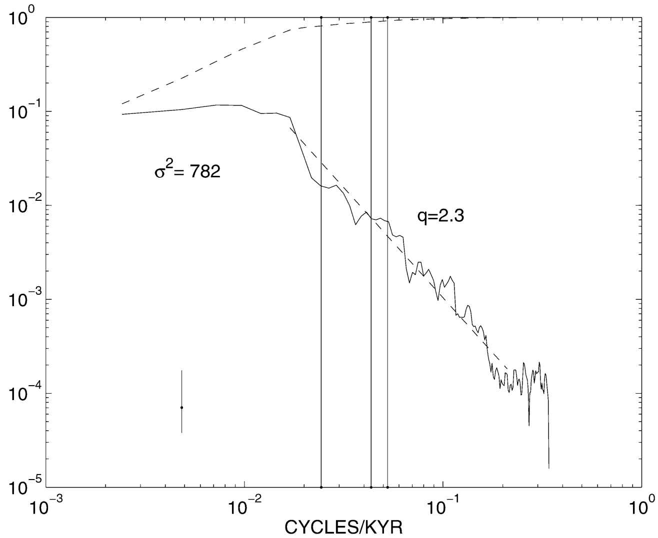


Fig. 10 Power spectrum of the Vostok record in Fig. 9. Note the white-noise character at the highest frequencies, suggestive of the sampling noise ("least-count") level

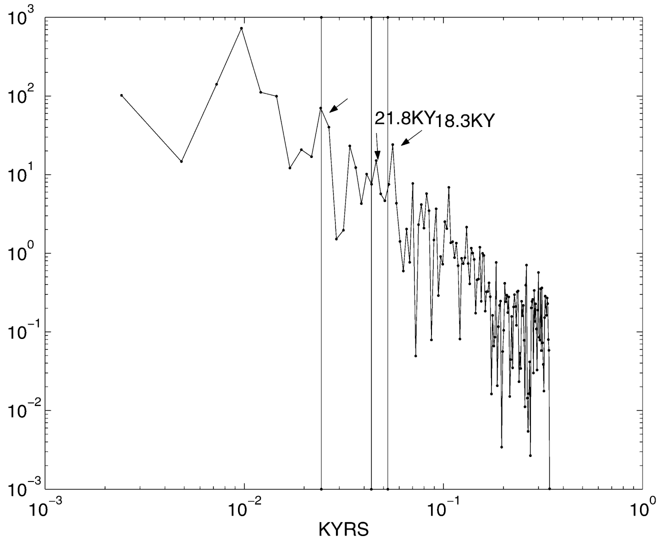


Fig. 11 Periodogram (not the power spectrum estimate) of the Vostok CO_2 record. *Arrows* indicate three apparent peaks which can be associated with the Milankovitch precessional and obliquity bands. The two precessional peaks differ slightly in period from those used to nominally delineate this band, and the estimated periods are shown. But precessional and obliquity bands together represent a small fraction of the total variance

Coincidence of a power law with that predicted by a simple memory process does not prove that the underlying physics has that character, but it is probably the simplest of all such rationalizations, and it would be very difficult to distinguish the time series from one following Eq. (4). The robust result is that the background continuum is the major descriptive feature of these records, and the Milankovitch contribution, where present, is a perturbation to the overall system, not the dominant forcing or response.

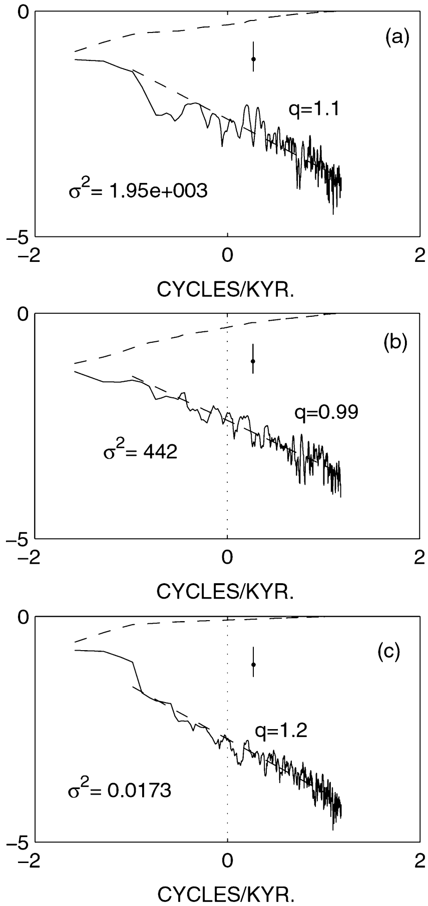


Fig. 12a–c Spectral estimates from core MD95-2006 in the Rockall Trough (Knutz et al. 2001). Shown are, (*a*) seismic velocity (p-wave), (*b*) magnetic susceptibility, (*c*) bulk density along with a best fitting power law in each case

3.2 The 100 ky problem-a stochastic time scale

After the general red-noise character of the spectra, the next most conspicuous feature is the relative bulge in energy at periods near 100,000 years, with there being a change in character (to nearly white) at longer periods. Much has been written about the 100 ky energy dominance and a number of ingenious theories has been constructed, mainly directed at finding an explanation in terms of a periodic response. These theories include the direct resonant response to the pure, weak, Milankovitch eccentricity forcing; non-linear and relaxation oscillations of the coupled ocean/atmosphere/cryosphere, including entrainment of a Milankovitch carrier; rectification of the higher frequency Milankovitch forcing, perhaps by excitation of a multi-state climate system (Paillard 1998); movement of the plane of the ecliptic relative to the invariable plane, among others. Roe and Allen (1999) examined many of the extant hypotheses about the 100 ky energy, and concluded that the data are inadequate to reject any of them.

Here, we ask whether it possible that the 100 ky dominance is also a primarily stochastic response? One might question the sense of adding yet another possibility to the already long list of mechanisms; the main justification is that most of the previous explanations have stipulated that the process is fundamentally a periodic one, and we explore the possibility of a non-oscillatory cause. (True periodic components are not ruled out; they are considered at least tentatively, to be small relative to the non-periodic energy.)

Consider, as a particularly simple stochastic framework, a climate system which becomes unstable and collapses when it reaches a critical part of its phase space. MacAyeal (1993), e.g., discusses such an instability in an icesheet as a possible explanation for the much shorter time scale Heinrich events. But here, unlike this and other discussions of an ice sheet contribution to producing the 100 ky time scale, *no oscillator is required*. Ice, or other elements of the climate system, accumulate under random forcing, until a stability threshold is crossed, when the system collapses, only to regrow as a random walk builds it up again. If left unforced in any state, there would be no change and hence this response is not an oscillator. For the ice component alone, the physics involves rheology and bottom stresses (MacAyeal 1993), and other processes (e.g., sea ice; see Gildor and Tziperman 2000). Invocation of an icesheet instability is only intended as an example; such an instability might be that of the climate system as a whole, with ice, ocean and atmospheric circulation being collectively in a state subject to sudden collapse (see e.g., Tarasov and Peltier 1997).

Although the situation need not be so simple, a useful question is how far one can go with this purely kinematic picture? The random forcing consists of all of the higher frequency fluctuations in the climate system that are the result of internal and external stochastic fluctuations. These would include ordinary weather processes, but also all of the myriad expected fluctuations of the coupled ocean/cryosphere/atmosphere/biosphere at periods shorter than 100 ky. The underlying stochastic driving can have a more complex spectrum than a white one, some contributions can even be deterministic, but the white-noise hypothesis is again the simplest possibility. Feedbacks in both the atmosphere and the ocean will lend color to the forcing process, but these, in the present simplistic approach, can be regarded as details. If a restoring "force" is present, so as to produce some true oscillatory behavior, this response can be superimposed without its having to be dominant.

As an exploratory example, we use Eq. (4) with $x(n)$, $a = 1$, $\Delta t = 1$, as a single variable surrogate for the climate state. In terms of a pure ice instability construct, it might represent the volume or elevation of the Northern Hemisphere icesheet, or that part grounded in the ocean (see Pisias and Moore 1981). To this simple expression, we append the requirement that if $x(n) < 0$, it is re-set to zero (no negative ice volumes; but any finite positive minimum representing, e.g., today's climate state, could be chosen), and if $x(n) > \xi_0$, it is again set to zero (an oversimplification of the rapid decay process, which could be represented instead by a finite duration collapse time). MacAyeal's (1993) unstable filling bucket remains a good analogue, although here it is regarded as representing the climate system as a whole, not as a local phenomenon describing Heinrich events.

To provide a slightly greater element of realism, we take $\xi_0 = \bar{\xi}_0 + r$, where r is again a random variable, with standard deviation of 10% of $\bar{\xi}_0$. This form says that the collapse threshold is not an exact value. In the example shown in Fig. 13, the time step is unity, $\bar{\xi}_0 = 20$, $<\theta^2> = 1$ (the bracket denotes an ensemble average). No attempt was made to "tune" $\bar{\xi}_0$, but visually, one sees a build up and decay of $x(n)$ on an approximately 100 time step scale, with a subsequent collapse to zero. There is evidently a connection among the values of $\bar{\xi}_0$, a, $<\theta^2>$, and the probability of encountering the threshold; this connection can be found through the theory of threshold-crossing statistics, the "gambler's ruin" problem, and random walk with reflecting barriers (Feller 1957), but it is not pursued here. Simple analysis however, shows that if one ignores the influence of the threshold variance parameter, r, that the expected time to encounter the instability limit should behave as,

$$t \sim \frac{\xi_0^2}{<\theta^2>} \ , \tag{7}$$

from the well-known result that the expected distance increase of simple random walk is proportional to $\sqrt{t}$. Thus if the white noise variance should diminish by a factor of 2, the apparent time scale would double.

We could make plausible more complex rules than Eq. (4) that would produce more complicated behavior, but perhaps the main point is already clear: the presence of a dominant time scale does not require a periodic oscillation and depends only upon having a sub-system with a memory, and a mechanism for ultimately

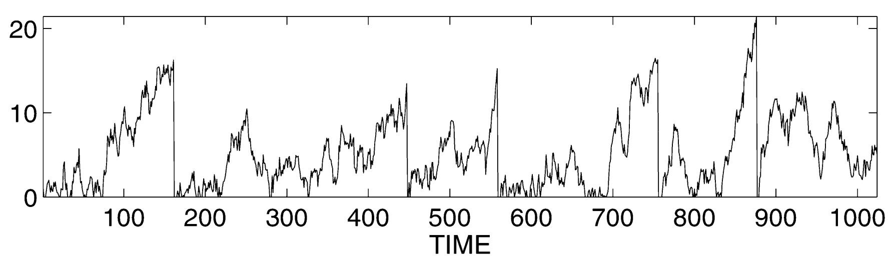


Fig. 13 Synthetic time series generated from a simiple random walk and a threshold collapse requirement. All units are arbitrary here. (For comparison e.g., with the Vostok CO_2 record, Fig. 9, the abcissa should be inverted)

destroying the memory. That the 100 ky cycle is not periodic is an old idea; Winograd et al. (1992) for example, show that the Devil's Hole record displays a nonperiodic result (see especially, their Fig. 3).

The power density estimate for $x(n)$ is shown in Fig. 14; it produces a qualitatively realistic maximum near 1 cycle/100 time units (again no attempt was made to tune the parameters to produce a near 100 time-unit peak, nor to control its shape). Because of the wiping out of the memory at the threshold, the spectrum has a different character at periods longer than the peak, as observed in the various core spectra. There are other, non-statistically significant, peaks, but the record is dominated by the s^{-2} rule at high frequencies. If one looks at the temporal behavior of a scalar process alone, the visual behavior of the time series and of the spectral shape are all qualitatively consistent with the spectra from both deepsea and ice cores.

Returning to the gambler's ruin/random walk analogy, one might think of this picture as equivalent to a "drunkard's climb." A drunk on a ladder randomly takes steps upward or downward. He cannot go below the floor. Once in a while, he mounts the ladder so far, that he overtops it, falling to the floor, where he immediately remounts and re-starts his random ascent/descent.

Coefficient a in Eq. (4), governing the accumulation of ice and changes in the atmosphere/ocean circulation, subsumes a great variety of physical processes. Many of the mechanisms discussed, e.g., by Tarasov and Peltier (1997), would be involved, as would all of the implied feedback terms. No one-dimensional kinematic model can capture such behavior, but in terms of the gross spectral behavior, the kinematic model is at least strongly suggestive that a dominant time scale can appear in a record with no oscillator present.

Figures 3 and 5 show that the 100 ky time scale coincides with a change in spectral slope, as much as it coincides with a spectral excess. It is worth noting that for a simple red-noise process, the *absence* of energy in a frequency band can produce a visual impression of a dominant time scale. Figure 15 was generated through the conventional AR(1) algorithm,

$$x_{n+1} = 0.99x_n + \theta_n \ , \tag{8}$$

where θ_n is unit variance white noise, and then notch filtered to remove all energy between $10^{-3} \leq s \leq 10^{-2}$ cycles/unit time by suppressing the Fourier series coefficients there. The resulting spectra in both linear and log-log form show, particularly in the linear form, an apparent peak near 1 cycle/100 time units, which however, is a reflection of the absence of energy at the adjacent lower frequencies. What time series like Eq. (8) lack is the strong asymmetry seen in the deglaciation part of the cycle, and which leads to the introduction of the hypothesis of an asymmetric collapse mode.

Nothing rules out the simultaneous presence of a pure forced motion at 100 ky, e.g., from the eccentricity components of Milankovitch insolation variations, or a mechanism such as proposed by Muller and Macdonald (2000) superimposed upon the background, although such coincidences are always suspect. But could the observed response be that of an internal oscillator excited by the random forcing?

The simplest oscillator is the ordinary linear one, analogous to a mass-spring system, e.g., satisfying an equation of form

$$m\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2} + r\frac{\mathrm{d}x(t)}{\mathrm{d}t} + k_0x(t) = q_c(t) \ . \tag{9}$$

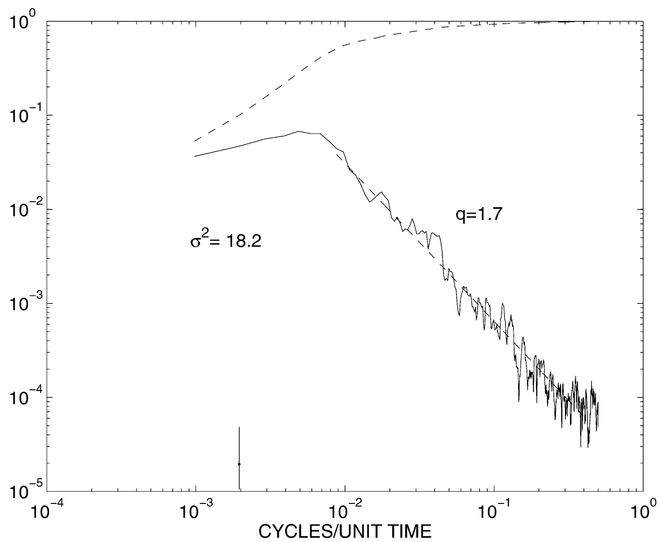


Fig. 14 Power spectral estimate of the record in Fig. 13 showing the high frequency power law ending near 10^{-2} cycles/time unit in a broad maximum with a decline at longer periods. An approximate 95% confidence interval is shown; none of the high frequency structures are statistically significant

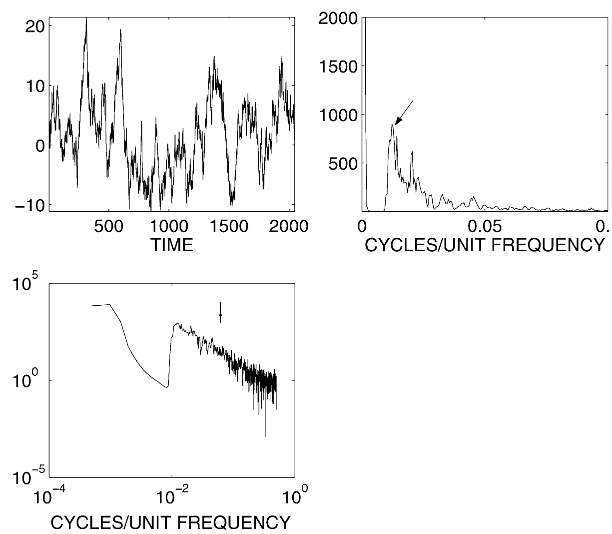


Fig. 15 A simple red-noise process (*upper left panel*) notch filtered to remove energy in a finite frequency band. *Upper right* and *lower left panels* display the resulting spectra in linear (the former), and logarithmic forms. No actual spectral peak exists

Here, m is a mass-equivalent, k_0 a spring constant, and r a damping coefficient. $q_c(t)$ is a driving function, which in continuous form would be a Wiener or Brownian-motion process. The natural frequency (with $r = 0$), is $s_0 = \sqrt{(k_0/m)}/(2\pi)$. Equation (9) is readily discretized as

$$\mathbf{x}((n+1)\Delta t) = \mathbf{A}\mathbf{x}(n\Delta t) + \mathbf{q}_c(n\Delta t), \quad n = 0, 1, \ldots \tag{10}$$

where Δt is a time-increment, $\mathbf{x}\ (n\Delta t) = [x(n\Delta t), x((n-1)\Delta t)]^T$, $\mathbf{q}_c\ (n\Delta t) = [(\Delta t)^2\ q_c\ (n\Delta t), 0]^T$ in standard vector notation, with the state-transition matrix,

$$\mathbf{A} = \begin{Bmatrix} 2 - k_0(\Delta t)^2 - r\Delta t & -1 + r\Delta t \\ 1 & 0 \end{Bmatrix} . \tag{11}$$

Choosing $\Delta t = 1$, $k_0 = 0.01$, $r = 0.01$, and taking $q_c(n\Delta t)$ to be unit variance white noise, one obtains a synthetic time series from Eq. (10). The estimated spectral density (easily determined analytically) is shown in Fig. 16. One sees a change in spectral shape across the near-resonant peak, corresponding to the near-static response of a mass-spring oscillator at frequencies $s \ll s_0$, and with a dynamic response otherwise. Notice that the high frequency slope of the spectral density is much steeper (s^{-4}) than what is observed in any real core. If q itself is a red-noise process, then it is simple to show that $\Phi(s)$ is even steeper (s^{-6} for an AR(1) process). It would appear then, that a hypothesis of a simple resonant mode near 100 ky cannot alone explain the spectral behavior at shorter periods, and that some other process must, at the very least, be present too.

3.3 The "pacemaker" hypothesis

As noted in the Introduction, a version of the hypothesis of control of climate by insolation variations stipulates

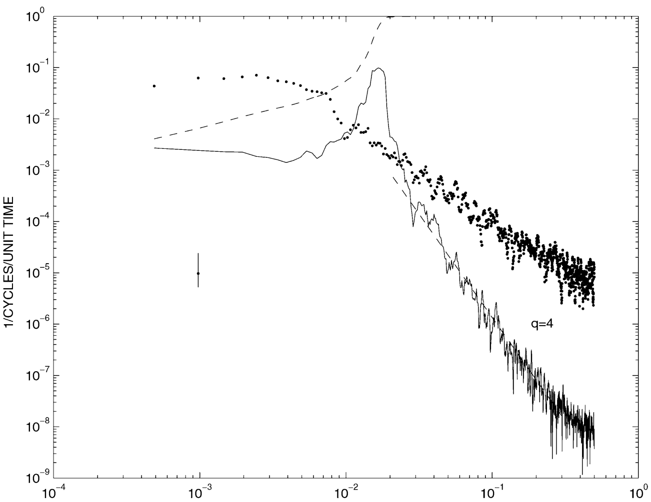


Fig. 16 Linear oscillator. $k_0 = 0.01$, $r = 0.01$, $\Delta t = 1$, driven by a white-noise stochastic process. All units are arbitrary. The high frequency rise to the resonant peak is proportional to s^{-4}. For comparison, the s^{-2} spectrum of a simple autoregressive process (AR(1), $a = 0.99$) is shown as *a dotted line*. At low frequencies, both spectra become nearly white

that the 100 ky glacial cycles are determined by the net levels of high latitude insolation through the low-frequency beating of obliquity and precessional terms. In one form of this view (A. Berger personal communication, 2002), the absence of strong responses in the obliquity and precessional bands is immaterial because the system is instrinsically non-linear. Elkibbi and Rial (2001) have recently summarized much of the debate surrounding this type of mechanism.

There is little doubt that climate change involves a myriad of non-linear processes, many of which have been discussed in the literature. Indeed, as Rubincam (1994) has pointed out, any real energy in the precessional band is necessarily the result of non-linear rectification of the modulation of the seasonal cycle, which vanishes in a yearly average. (Huybers and Wunsch in preparation 2002, have suggested, that in a nonlinear system, it would be very difficult to distinguish direct precessional-forced response from simple first harmonics of an obliquity-band response.)

The most familiar non-linear processes are ones for which periodic, or narrow-band, driving produces a measurable response at the driving frequency, as well as at the overtones and sum and difference frequencies of the driving. Often, the fundamental, or linear, response is the dominant one, and one expects that to be the case in any weakly non-linear system. For example, typical non-linearities of the Navier-Stokes equations governing the oceanic and atmospheric elements of climate, involve terms such as $u\partial u/\partial x$, where u is a velocity component, and x a coordinate. Such quadratic non-linearities, even in strongly driven periodic flows like tidal streams, do not destroy the powerful motions at the driving frequencies. Very general results for both stochastic and periodic driving are available for this and similar systems (e.g., Middleton 1960).

The common existence of this type of nonlinear system does not of course, preclude the possibility of systems which are so non-linear that the fundamentals and simple harmonics are suppressed; many of the multi-state climate models have this character. Nonetheless, such models are not readily derivable from the known equations of motion, and in particular, they suppress (or parameterize) the representation of finer scale elements which would be expected to show the linear response. But no claim is made to have disproven the possibility of a non-linear climate system without a strong response at the forcing fundamentals. Furthermore, it is conceivable that the vagaries of what the cores do record suppresses high frequency elements, while displaying only the much lower frequencies (filtering).

Setting aside however, any concerns about the absence of a strong direct Milankovitch response in the record, is it nonetheless possible that the glacial-interglacial cycles are controlled by the higher frequency insolation forcing? This question will not be answered here, leaving it as one of the many extant explanations of the 100 ky time scale. Two elements do however, immediately impinge on the answer: (1) cores with age-models tuned to the Milankovitch bands have suspect phase relationships (see Huybers and Wunsch 2002), and (2) in dealing with oscillatory records, one must evaluate the probability of false positive results. To my knowledge, problem (2) has not been studied.

4 Aliasing issues

For practical reasons, most cores, either marine or ice, are sampled comparatively infrequently. The phenomenon of aliasing is the spurious appearance of high frequency energy at low frequencies, when the record is not sampled rapidly enough. A mathematical statement is that to avoid the effect, one must sample at (uniform) time intervals,

$$\Delta t < 1/(2s_{\max}) \ , \tag{12}$$

where $s_{\max}$ is the highest frequency contributing to the underlying continuous record. See the discussions, e.g., by Pisias and Mix (1988) and Wunsch (2000).

The rigorous requirement can never actually be met with a finite duration record, but in practice all one need require is that the energy inappropriately occurring at low frequencies should be an acceptably small fraction of the energy which properly belongs there. If a spectral density diminishes rapidly enough with frequency above some frequency, s_0, one can simply choose $\Delta t < 1/(2s_0)$ and the residual aliasing will be negligible.

If the spectral shape overall is proportional to an s^{-2} or steeper law, one can (e.g., Wunsch 1972) choose Δt almost arbitrarily, simply because the energy rise at low frequencies is so steep that no degree of aliasing produces more than a negligible amount of aliased power. For power law spectra less steep than –2, one needs to be careful.

One must thus be wary of accepting the accuracy with which the spectral density power laws have been estimated here: they may well be inaccurate. In particular, estimates displayed in some of the figures show approximate s^{-1} behavior to the highest frequency estimated and there is no indication that the drop-off at frequencies above $1/(2\Delta t)$ is any more rapid. That is, one hopes to see the spectral density estimate show a roll-off faster than s^{-2} as the highest estimated frequency is approached, implying that adequate sampling has taken place. We do not see this behavior with some of the core data (e.g., MD95-206). That the inferred power laws are inaccurate is a reasonable, if undemonstrable, inference.

D. Gunn (private communication 2001) has resampled part of core MD95-206 at 20 times finer depth intervals than the original 2 cm sampling used to generate Fig. 12. A preliminary analysis (details to be published elsewhere) shows a qualitative change in behavior at the higher sampling rates, and a significant aliasing of energy to low frequencies, especially in the bulk density estimates, and hence a change in spectral shape as well as in apparent temporal behavior.

Assuming for the moment however, that the power laws displayed in the various figures here are approximately accurate, they show $0.8 \leq q \leq 2$. The steeper rules, $q \approx 2$ (e.g., the Vostok CO_2 record), are consistent with the simplest memory process (Eqs. 5 or 4). The shallower rules require a somewhat more complex system, e.g. through the addition of extra terms in $x((n-r)\Delta t)$, $r > 1$ to Eq. (4) or introduction of vector-valued $x(n\Delta t)$ representing spatial degrees of freedom, or both, or the introduction of bioturbation and other processes. But the principle, that one can generate power laws through simple rules, is not in doubt. The reader is reminded that for spectral rules less steep than $q = 2$, that the spectrum of the time-rate of change of the underlying variable is dominated by the *highest* frequencies present.

5 Summary and discussion

Much of climate change out to periods of 100,000 years (and probably longer), is describable as a stochastic variable, often associated with random walks, as discussed much earlier by Kominz and Pisias (1979). Beyond 100 ky periods, indications are that the spectrum becomes white until much longer time scales are reached, and at shorter periods, it is red-noise. Detailed characterization of the process in some cases is uncertain owing to questions about the adequacy of sampling of the available cores. Most theories relate the time rates of change of climate variables such as temperature or ice volume to forcing functions, and unless the spectral densities are very steeply diminishing functions of frequency, the physics will be governed by the highest frequencies present, not the lowest.

Superimposed upon some of these spectra are weak structures corresponding to the major Milankovitch forcing frequency bands, but none of the records examined here could be said to be dominated by the Milankovitch periods. Mitchell (1976) has a good general discussion of the spectral components of climate change.

The 100 ky changes may have a periodic component, but they are not obviously controlled by that behavior. Stochastic forcing of a system with a collapse threshold can display a variability not unlike that observed for the 100,000 year glacial-interglacial oscillation without any oscillator being present. The shifts are non-periodic, yet display a visually dominant time scale. Such oscillations are not predictable, except in the statistical sense that the mean time to a threshold crossing can be calculated (Feller 1957).

Reduction of climate system behavior to the simple buildup to a threshold and collapse, is obviously a gross oversimplification. Nothing precludes the co-existence of other modes of instability or of true oscillations or both, or the addition of determinstically (Paillard 1998), or randomly excited multiple states; in general the climate system as a whole would be involved. One anticipates both regional and global contributions from ice sheet, ocean and atmosphere movements. But no single time scale or process seems dominant (except for the 100 ky mode). To the extent that other instabilities of the system exist, they could well produce other (Heinrich event-like) fluctuations regionally and globally. Alternatively, a reduction (increase) of the stochastic variability would lengthen (shorten) the dominant time scale from the present 100 ky one. It is possible that the change in power law in the spectra around periods of 100 ky is also associated with a true oscillator peak at that time scale; but the observed slope change is not generated by a simple oscillator alone.

With several competing hypotheses all purporting to explain the 100 ky structure of the spectrum, attention needs to turn to devising rigorous tests for distinguishing them in real records. Some tests are fairly straightforward; for example, mechanisms requiring non-linearities should, conventionally, generate overtunes in the spectrum as well as undertones. Absence of those overtones does not destroy the hypothesis, but their presence would greatly strengthen it. Similarly, with long enough records, one can begin to distinguish continuum energy from line energy and incoherent energy from the part coherent with insolation forcing. Linear and non-linear systems tend to have different probability density functions. These and other possibilities are left for future work.

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